

On the possible mechanism to form the radio emission spectrum of the Crab pulsar

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Accepted 2014 March 07

ABSTRACT

In the present paper a self-consistent theory, explaining shape of the observed phase-averaged radio spectrum in the frequency range from 100MHz to 10GHz is presented. The radio waves are assumed to be generated near the light cylinder through the cyclotron resonance. The cyclotron instability provides excitement of the electron-positron plasma eigen-waves, which come in radio domain when the resonant particles are the most energetic primary beam electrons. It is widely accepted that the distribution function of relativistic particles is one-dimensional at the pulsar surface. The generated waves react back on the resonant particles causing their diffusion in the perpendicular direction to the magnetic field and violating the one-dimensionality, which switches on the synchrotron radiation process. The synchrotron emission of the beam electrons provides generation of high-energy γ -rays simultaneously with the radio emission, that explains the observed pulse phase-coincidence in these energy domains. The theory provides a power-law radio spectrum with the spectral index equal to 3.7, coming in agreement with the observations.

Key words: Pulsars: individual: PSR B0531+21 – Radiation mechanisms: non-thermal.

1 INTRODUCTION

The only energy source providing the observed emission of Crab nebula and pulsar, is the pulsar rotation slowdown energy. Thus, one of the main problems in pulsar emission theory is to find the transformation mechanism of the rotational energy into the observed radiation. Due to works Goldreich & Julian (1969); Sturrock (1971) and etc. a well-defined scheme of serial processes has developed, which provides the solution of the mentioned problem. In particular, we believe that at the polar cap due to rotation of the magnetized neutron star the electric field is generated, which extracts the primary beam of electrons from the pulsar surface and accelerates them. Moving along the weakly curved magnetic field lines the beam electrons start to generate γ -quanta. As long as the energy of gamma-quanta in the laboratory frame satisfies the condition $\varepsilon_\gamma \sin \alpha > 2mc^2$ (here α is the angle between the magnetic field and the direction of motion of the γ -quanta) in the strong pulsar magnetic field develop the quantum effects of cascade creation of electron-positron (e^-e^+) pairs (Ruderman & Sutherland 1975; Michel 1982). The size of the region near the star surface where the longitudinal component of the electric field

is nonzero (gap region) does not exceed few meters. After leaving the gap region charged particles move along the magnetic field lines, as they loose their perpendicular momenta in a very short time ($\leq 10^{-20}$ s) due to the synchrotron radiation processes. Thereby, the magnetospheric e^-e^+ plasma is formed, with an anisotropic one-dimensional distribution function (see Fig.1 from Arons (1981)), in which generate the electro-magnetic waves leaving the pulsar magnetosphere and reaching the observer as the pulsar emission.

The Crab pulsar differs from the most of radio pulsars, as emits in a broad range, from radio up to very high energy gamma-rays. The important observational feature of the Crab pulsar emission is, that its multiwavelength radiation pulses are coincident in phase (Manchester & Taylor 1980; Aliu et al. 2011). This feature should indicate that the emission from different energy bands is generated simultaneously at the same location in the pulsar magnetosphere. In our previous works (Chkheidze et al. 2011, 2013) was considered the simultaneous generation of the high-energy γ -rays from 10MeV to 400GeV, and the radio waves at the light cylinder length-scales in the magnetosphere of the Crab pulsar. The reason for the wave generation in the outer parts of the pulsar magnetosphere is the one-dimensionality and anisotropy of the particles' distribution, which leads to development of plasma instabilities. In particular, in the Crab pulsar's

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magnetosphere near the light cylinder the conditions for the cyclotron resonance are satisfied, which provides generation of the low frequency cyclotron waves (Kazbegi et al. 1992). The resonant particles diffuse in both directions along and across the magnetic field during the quasi-linear relaxation stage of the instability and acquire the perpendicular momenta. Consequently, the particles start to radiate through the synchrotron mechanism. We assumed that the resonant particles are the most energetic primary beam electrons with the Lorentz factors $\gamma \sim 10^7 - 10^9$, which provides the synchrotron radiation of energetic γ -rays up to 400 GeV. Such high energies by beam electrons are reached during their re-acceleration in the light cylinder region due to the Landau damping of electrostatic waves (see Mahajan et al. (2013); Chkheidze et al. (2013) for details). At the same time such particles can provide generation of the cyclotron modes in the radio domain. In previous work (Chkheidze et al. 2013) we only considered the theoretical explanation of the measured γ -ray spectrum of the Crab pulsar and explained coincidence of γ -ray and radio signals.

Consequently, in the present paper we consider generation of radio waves via the cyclotron resonance in more details (Sect. 2), calculate the theoretical radio-spectrum (Sect. 3) and make conclusions (Sect. 4).

2 GENERATION OF RADIO EMISSION

Differently from gamma-ray emission, which does not 'feel' the medium and emerges freely from the region of its generation, for the low frequency radiation the condition is satisfied $\lambda \gg n^{-1/3}$ in the pulsar magnetosphere (here n is the density of plasma particles, and λ is the wave-length of low frequency waves). In this case one should take into account the effects of wave interference, which means applying the plasma physics methods. Such problem was considered for the cold plasma in hydrodynamical approach in works (Buschauer & Benford 1977; Hardee & Rose 1976). And for the plasma with an arbitrary temperature this problem was solved in kinetic approach, in the following works Volokitin et al. (1985); Arons & Barnard (1986); Lyutikov (1998).

It is usually suggested that the pulsar magnetosphere is filled by relativistic e^-e^+ plasma. In strongly magnetized cold e^-e^+ plasma can be excited only waves propagating almost along the magnetic field lines (Kazbegi et al. 1991). As we have already mentioned during the pair creation near the star surface, the perpendicular momenta of charged particles are instantly lost through the synchrotron radiation, making the particle distribution one-dimensional. The plasma with such distribution is unstable and excites plasma eigen-waves (the so called t -waves) at a certain resonance condition (Lominadze et al. 1983). The t -wave is a purely transverse wave with an electric field perpendicular to (\mathbf{k}, \mathbf{B}) plane, where \mathbf{k} is the wave vector and \mathbf{B} is the vector of the magnetic field. The dispersion relation of the t -mode is written as

$$\omega^t = kc(1 - \delta), \quad (1)$$

where $\delta = \omega_p^2 / (4\omega_B^2 \gamma_p^3)$, $\omega_B = eB/mc$ is the cyclotron frequency and the square of the plasma frequency $\omega_p^2 = 4\pi n_p e^2 / m$.

The resonance condition of the cyclotron instability which can provide generation of t -waves can be easily obtained, if considering the difference between the quantum synchrotron levels l and f (Jelezniakov 1977; Ginzburg 1981)

$$E_l - E_f = (m^2 c^4 + p_l^2 c^2)^{1/2} - (m^2 c^4 + p_f^2 c^2)^{1/2} = \hbar \omega_{lf}, \quad (2)$$

where $p^2 = p_{\parallel}^2 + p_{\perp}^2$. Taking into account that $m^2 c^2 / p^2 \approx 1/\gamma^2 \ll 1$ and $p_{\perp}^2 / p_{\parallel}^2 \ll 1$ are the small parameters, one can expand Eq. (2) in series and obtain

$$(p_{\parallel l} - p_{\parallel f}) v_{\parallel} + \frac{1}{2m\gamma} (p_{\perp l}^2 - p_{\perp f}^2) \approx \hbar \omega_{lf}. \quad (3)$$

Assuming, that $p_{\parallel l} - p_{\parallel f} = \hbar k_{\parallel}$ and $(p_{\perp l}^2 - p_{\perp f}^2) / (2m) = \hbar s \omega_B$, where $s \equiv l - f$ denotes the difference between the synchrotron Landau levels l and f , we will get

$$\omega_r - k_{\parallel} v_{\parallel} - s \frac{\omega_B}{\gamma} = 0. \quad (4)$$

It is clear that, when $s = -1$ (the anomalous Doppler effect) the wave energy grows at the expense of longitudinal energy $\omega_r k_{\parallel} v_{\parallel} < 0$, i.e. $v_{\parallel} > v_{ph}$ (here $v_{ph} = \omega/k$ is the phase velocity of excited waves) and waves gain particles energy through the interaction process. The waves, as well as the particles leave the pulsar magnetosphere in time of the order of $\Omega^{-1} \approx 10^{-2}$ s (here $\Omega = 2\pi/P$, and P is the pulsar spin period). This does not prevent the instability process, as the resonance area is always filled with newly arrived particles which take the place of particles that leave this region. The frequency of excited t -waves can be estimated from Eq. (4) as follows (Chkheidze et al. 2013)

$$\omega_r \approx \frac{\omega_B}{\delta \gamma}. \quad (5)$$

As we see the frequency of generated t -waves depends on the Lorentz-factors of the resonant particles, the higher is the energy of the particles the lower is the frequency of the excited eigen-waves. For the most energetic particles in the pulsar magnetosphere, the primary beam electrons (which Lorentz factors vary from 10^7 to 10^9 , for PSR B0531+21, as assumed in Chkheidze et al. (2013)) the frequency of the transverse waves come in the radio domain. In particular, it varies in the frequency range from 100 MHz to 10 GHz.

Under influence of the electric field of the generated radio waves the resonant particles diffuse along and across the magnetic field lines (Kazbegi et al. 1991). This causes the particles' redistribution and the distribution function acquires the perpendicular component by impulses. The growth of the perpendicular impulses continue till the balance is achieved between the processes of pumping the longitudinal energy of particles into their transversal motion and the energy losses caused due to their synchrotron radiation. This process can be described by the following equation in kinetic approach (see Eq. (11) in Chkheidze et al. (2013))

$$\frac{\partial f^0}{\partial p_{\perp}} = \frac{F_{\perp}}{D_{\perp, \perp}} f^0. \quad (6)$$

where $D_{\perp, \perp} = e^2 / (8c) \delta |E_k|^2_{k=k_{res}}$ is the transversal diffusion coefficient, $|E_k|^2$ is the density of electric energy in the excited waves and F_{\perp} is the transversal component of the synchrotron radiation reaction force, which has the form

$$F_{\perp} = -\alpha_s \frac{p_{\perp}}{p_{\parallel}} \left(1 + \frac{p_{\perp}^2}{m^2 c^2} \right), \quad \alpha_s = \frac{2e^2 \omega_B^2}{3c^2}. \quad (7)$$

From Eq. (6) one can easily find

$$f(p_{\perp}) \propto e^{-\left(\frac{p_{\perp}}{p_{\perp 0}}\right)^4}, \quad p_{\perp 0} = \left(\frac{4\gamma_r m^3 c^3}{\alpha_s} D_{\perp, \perp}\right)^{1/4}. \quad (8)$$

During the quasi-linear stage of the resonance the excited waves cause particle diffusion as along also across the magnetic field lines. The transversal diffusion leads to appearance of the perpendicular impulses and the one-dimensional distribution function acquires the transversal component. The particles emit in the synchrotron regime when having the perpendicular component of the impulse. The characteristic frequency of the synchrotron radiation of a single electron is $\epsilon_{syn} \approx 5 \cdot 10^{-18} B \psi \gamma^2 \text{ GeV}$ (Ginzburg 1981). The energy of the synchrotron photon reaches $\epsilon_{syn} \approx 400 \text{ GeV}$, if the Lorentz factor of the emitting particles is of the order of 10^9 . The particles' acceleration mechanism up to Lorentz factors of the order of 10^9 is considered in previous work Chkheidze et al. (2013).

The synchrotron radiation reaction force plays the main role in the process of wave generation and redistribution of the resonant particles. In particular, the transversal component of the synchrotron emission force F_{\perp} confines the growth of the pitch angles and the longitudinal component $F_{\parallel} = -\alpha_s \gamma_r^2 \psi^2$ redistributes the emitting particles by their parallel momenta (this force appears to be stronger than the diffusion forces). The longitudinal distribution function of the resonant particles after the quasi-linear relaxation takes the following form (see Eq. (27) in Chkheidze et al. (2013))

$$f_{\parallel} \propto \frac{1}{p_{\parallel}^{1/2} |E_k|}. \quad (9)$$

The spectral density of the excited waves $|E_k|^2$ depends on the initial distribution function of the emitting particles (for the initial moment is chosen the moment when the wave excitation starts). At the initial moment the distribution of the beam particles can be presented as (Chkheidze et al. 2013)

$$f_{\parallel 0} \propto \left\{ \begin{array}{l} p_{\parallel}^{-n}, \quad p_{\parallel \min} \leq p_{\parallel} \leq p_{\parallel s} \\ p_{\parallel}^{-m}, \quad p_{\parallel s} \leq p_{\parallel} \leq p_{\parallel \max} \end{array} \right\}. \quad (10)$$

Here $p_{\parallel \min} = mc^2 \gamma_{b \min}$, $p_{\parallel s} = mc^2 \gamma_{b s}$, $p_{\parallel \max} = mc^2 \gamma_{b \max}$ and it is assumed that $\gamma_{b \min} \simeq 6 \cdot 10^6$, $\gamma_{b s} \simeq 4 \cdot 10^7$ and $\gamma_{b \max} \simeq 10^9$. One can easily represent the function (10) by the following expression

$$f_{\parallel 0} \propto p_{\parallel}^{-n} + \beta p_{\parallel}^{-m}, \quad (11)$$

where $\beta \simeq 1.3 \cdot 10^{-10}$ defines the break point of the initial distribution between two power-laws. The power-law with the index m corresponds to re-accelerated beam particles near the light cylinder via the Landau damping (see Chkheidze et al. (2013) for details). Referring to the Eq. (36) from Chkheidze et al. (2013)

$$a \frac{\partial}{\partial p_{\parallel}} \left(\frac{|E_k|}{p_{\parallel}^{1/2}} \right) + f_{\parallel 0} = 0, \quad (12)$$

one can find the energy density of the cyclotron waves using the Eq. (11) for $f_{\parallel 0}$, which writes as

$$|E_k| \propto p_{\parallel}^{3/2-n} + \beta \frac{n-1}{m-1} p_{\parallel}^{3/2-m}. \quad (13)$$

Thus, for the longitudinal distribution function of the emitting particles after the stationary state is reached from Eq. (9) we will get

$$f_{\parallel} \propto \left(p_{\parallel}^{2-n} + \beta \frac{n-1}{m-1} p_{\parallel}^{2-m} \right)^{-1}. \quad (14)$$

The redistribution of the resonant particles via the synchrotron radiation reaction force influences the process of excitement of the t -waves. As the waves are generated on account of the longitudinal energy of particles with the distribution function given by Eq. (14).

3 THE SPECTRUM OF THE RADIO EMISSION

For easy understanding of the formation of spectrum of excited t -waves we use the quantum mechanical approach. Let us consider two energy levels l and f ($l > f$). The number of particles at l level is N_l and N_f is the number of particles at the energy level f . Through the decay of particles from a higher energy level (l) to a lower one (f) the radio waves are generated. The process can be described by the coefficient A_f^l , which gives the probability per unit time that the particle in state l will decay to state f . The quantity $A_f^l N_l$ in this case shows the mean value of transitions from state l to state f per unit time. At the same time particles fill the energy level l , as they interact with the electromagnetic field of the waves with the frequency of the transition between energy levels l and f . It should be mentioned that particles which provide the generation of the observed radio emission also take place in the synchrotron emission process, as through the cyclotron resonance they acquire pitch angles. The synchrotron mechanism generates high-frequency radiation that freely leaves the magnetosphere. The only influence of this process on the excitement of radio waves is shown via the synchrotron radiation reaction force that provides redistribution of the resonant particles (see Eq. (14)), which interact with electromagnetic field of radio waves and fill the energy level l . The mean value of such transitions can be given by the following quantity $B_l^f N_f u(\nu)$, where $u(\nu)$ is the spectral energy density of the radiation field at the frequency of transition. The coefficient B_l^f is the probability per unit time per unit spectral energy density of the radiation field that particle at state f will jump to state l . The difference between the number of particles of populated energy levels is constantly maintained, due to arrival of the new particles in place of ones leaving the pulsar magnetosphere. Consequently, the energy loss should not be taken into account. Now if we equal the numbers of transitions between energy levels l and f that define the radio domain by writing

$$A_f^l N_l = B_l^f N_f u(\nu), \quad (15)$$

we will obtain the expression for the spectral energy density of the radio emission, which writes as

$$u(\nu) = \frac{A_f^l N_l}{B_l^f N_f}. \quad (16)$$

Here we have not taken into account the stimulated transition of particles from level l to f , as it does not play a role in radio-wave generation. Such transitions correspond

to generation of γ -rays through the synchrotron radiation. The ratio of probabilities of photon emission A_f^l and absorption B_f^l for any pair of levels l and f can be described by $|E_k|^2$, which can be easily found from Eq. (13). And the relation between the transition numbers N_l/N_f is given by Eq. (14). If we use the following expression $p_{||} = \hbar k$ in place of Eq. (16) we can write

$$u(k) \propto \frac{(k^{3/2-n} + \beta(n-1)/(m-1)k^{3/2-m})^2}{k^{2-n} + \beta(n-1)/(m-1)k^{2-m}}. \quad (17)$$

The shape of the radio spectrum can be defined after we give values for n and m . In previous work (Chkheidze et al. 2013), we showed that for explaining the observed γ -ray spectrum of the Crab pulsar up to 400 GeV one should assume that $n = 6$ and $m = 4.7$. In the frequency range from 100 MHz to 10 GHz, where the Crab pulsar's radio emission is detected the model spectrum defined by Eq. (16) behaves like $u(\nu) \propto \nu^{-3.7}$ for the above given values of n and m . This value is close to average value of the measured radio spectral index of the Crab pulsar in the given frequency range, which we define as $(\alpha_{MP} + \alpha_{IP})/2 \approx -3.6$, where the values of spectral indices for main pulse and interpulse are $\alpha_{MP} = -3.0$ and $\alpha_{IP} = -4.1$ (see Fig. 6 from Moffett & Hankins (1999)).

4 CONCLUSIONS

The emission generation mechanism in the outer parts of pulsar magnetosphere suggested in the present work provides explanation of the interesting observational feature of the Crab pulsar, the pulse-phase alignment of γ -ray signals in the energy domain (0.01 – 400) GeV with the radio signals. This behaviour is provided due to simultaneous generation of emission in the high and the low frequency ranges at the same location in the pulsar magnetosphere. To assure the validity of the proposed emission model, one should also explain the measured radio and γ -ray spectra. In work Chkheidze et al. (2013), we calculated the theoretical spectrum of the Crab pulsar in the 10 MeV to 400 GeV energy domain, which was presented by the following function $F(\nu) \propto [(\nu/\nu_0)^{-(n-2)/(n-4)} + (\nu/\nu_0)^{-(m-2)/(m-4)}]^{-1}$, where $\nu_0 \approx 9.7 \cdot 10^{23}$ Hz (this corresponds to photon energy $\epsilon_0 \approx 4$ GeV and shows the break point on the spectrum). To match the theoretical spectrum with the measured one best described by a broken power-law of the form $[(\nu/\nu_0)^{-1.96} + (\nu/\nu_0)^{-3.52}]^{-1}$ (Aliu et al. 2011; Aleksić et al. 2012), we assumed that $n = 6$ and $m = 4.7$. In present work we obtained the theoretical radio spectrum of the following form (see Eq. (17))

$$F(\nu) \propto \frac{((2\pi/\lambda)^{(m-n)} \nu^{3/2-n} + \beta(n-1)/(m-1) \nu^{3/2-m})^2}{(2\pi/\lambda)^{(m-n)} \nu^{2-n} + \beta(n-1)/(m-1) \nu^{2-m}}.$$

After using the values for n and m defined from the high energy spectrum, we found that in the radio domain, from 100 MHz to 10 GHz, the spectral function shows the simple power-law behaviour $F(\nu) \propto \nu^{-3.7}$.

The theoretically inferred value of the spectral index is close to the index of the radio emission phase-averaged spectrum, which equals 3.6. It should be mentioned that the radio spectral indices for the main and interpulse differ.

Particularly, index of the main pulse equals to 3.0, when the interpulse reveals the steeper radio spectrum with the index equal to 4.1 (Moffett & Hankins 1999). This observational fact might be caused by different reasons. One of the possible explanations could be the difference in distribution functions of particles responsible for the radio emission generation. This is very likely if the Crab pulsar is an orthogonal rotator, then the main and interpulse should be emitted from different poles of the star. Generation of particle beam with exactly the same distribution function in this case would be practically impossible.

It would be interesting to estimate the power of the radio emission. For this one can write the condition for conservation of quanta number, which has the following form

$$\frac{|E_k|^2}{\omega_k} = \text{const.} \quad (18)$$

Consequently, for the relation of gamma-ray and radio emission wave energy densities one can write

$$\frac{|E_k|_\gamma^2}{|E_k|_r^2} = \frac{\omega_\gamma}{\omega_r}. \quad (19)$$

After substituting the values for radio and gamma frequencies from Eq. (5) and Eq. (17) from (Chkheidze et al. 2013), one finds that $\omega_\gamma/\omega_r \simeq (10^6 - 10^8)$. Taking into account that the measured power of high energy emission of the Crab pulsar is of the order of 10^{36} erg/s, one can estimate the approximate power for the radio emission which from Eq. (19) follows to be of the order of $10^{28} - 10^{30}$ coming in a good agreement with the measurements.

ACKNOWLEDGMENTS

The research of the authors was supported by the Shota Rustaveli National Science Foundation grant (N31/49).

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